SHAPE RECOGNITION METHOD BASED ON THE k-NEAREST NEIGHBOR RULE

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Abstract: In artificial intelligence, each shape is represented by the vector of the characteristic features and represents a point in the d-dimensional space of the descriptors. An unknown shape is rejected or is identified with one or more models (known shapes previously learned). The shape recognition is based on the similarity between the unknown shape and each model. The distance between the associated two vectors is used for estimate the similarity between any two shapes. The paper presents a recognition method based on the k-nearest neighbors rule. This method supposes two stages: model classification and shape identification. Only invariant descriptors are used for the model classification at many levels and this classification is realized only once, in the learning stage of the recognition process. The shape identification supposes the identification of the most similar class of models, the identification of the nearest neighbors and the identification of the most similar model. A shape is identified with a model if that shape is inside the region of the accepted tolerance of the model. The proposed method assures fast and simple shape recognition in robotics.

Key words: shape, model, descriptor, recognition, classification, identification.

1 INTRODUCTION

Each shape x can be described by its vector of the characteristic features, $x = [x_1 x_2 ... x_d]^T$, and it represents a point $X(x_1, x_2, ... x_d)$ in the d-dimensional space of the descriptors.

An unknown shape can be identified with one or more models (known shapes or prototypes that form the training set). The shape identification is based on the analysis of the shape similarity (Purcaru 2003).

The most simple and intuitive method for the shape classification and identification is that based on the distance. Many distances are presented in (Dougherty 1988, Belaïd 1992, Kunt 2000, Purcaru 1999), and the choice of the proper distance depends on many factors:

- performances imposed to the shape recognition process;
- similarity between the models;
- recognition procedure.

An adequate reference distance is very difficult to find because a value too small sometimes determines the rejection of the shapes identical with some learned models, and the shape discrimination capability decreases if the value of the reference distance is too large.

Many methods are recommended in (Belaïd 1992, Kunt 2000) for the shape classification and decision:

- functional discrimination,
- automatic classification,
- Bayes classification procedure,
- k-nearest neighbor rule,
- stochastic methods,
- structural methods etc.

2 THE k-NEAREST NEIGHBORS RULE

The k-nearest neighbor rule (k-NN) was introduced by Fix and Hodges in 1951. According to this non-parametric procedure, an unknown shape x is assigned to the class that contains the majority of its k-nearest neighbors in the training set (Hastie 1996).

The main advantages of this classification method are the following:

- good performance in practical applications;
- remarkable properties for convergence, if the number of the models that form the training set tends to infinity;
- the identification of the class that contains x does not suppose complete statistical knowledge regarding the conditional density functions of each class (Denoeux 1995);
- if err_B is the error of the Bayes decision and err_{k-NN} the error of the k-NN decision,

$$err_B \le err_{k-NN} ... \le err_{2-NN} \le err_{1-NN} \le 2 \cdot err_B \quad (1)$$

and

$$\lim_{k \to \infty} err_{k-NN} = err_B. \quad (2)$$

When all the classes cannot be assumed to be represented in the training set, it may be wise to consider that a shape that is far away from any previously observed pattern, most probably belongs to an unknown

class and that shape should be rejected (Dubuisson 1993). Dasarathy has introduced in (Dasarathy 1980) the concept of "acceptable neighbor" that is the neighbor whose distance to the unknown shape (that must be classified) is smaller than a threshold value, learned from the training set.

The first version of k-NN method is considered slow for the shape identification because all possible distances must be computed between the unknown shape and each model, in the d-dimensional descriptor space. Many authors proposed different faster versions that can be grouped in four categories (Belaïd 1992), depending on their principle:

- the "compression" of the descriptor space dimension;
- the "paving", when the descriptor space is divided in cells and the study is limited to some cells;
- the "sorting", when the models are separated by the axes of the descriptor space;
- the "hierarchy", that supposes different levels of delicacy in the study of the models.

All these improved versions decrease the number of the analyzed models and, in this way, the shape identification time decreases too. Each technique imposes first a pre-processing stage of the training set. The efficiency of the method decreases when the dimension of the descriptor space increases.

The Kim and Park classification algorithm, presented in (Belaïd 1992), is based on the "sorting" principle; all the known shapes (models) $m_i = [m_{i,1} m_{i,2} .... m_{i,d}]^T$, $i = \overline{1, p}$ are sorted and form a tree with L levels of classification ($L_{max} \leq d - 1$). The sorting at each level is realized depending on the values of one descriptor. Let denote c_i, $i = \overline{1, L}$ the division factor of the value domain of the descriptor used for the level i of classification. There is minimum one node at this level and c_i branches leave from this node.

The algorithm of Kim and Park establishes the optimal path, in the tree of the known shapes, for finding the nearest (the most similar) neighbor of an unknown shape. The maximum number of comparisons for the shape identification is drastically reduced using this algorithm.

An analysis of the maximum number of distances (that must be computed for the shape identification) is presented in Table 1. This number is $N_C = P$ when the unknown shape is successively compared with each model m_i, $i = \overline{1, p}$. The classification based on the algorithm of Kim and Park reduces at N_C^* the maximum number of computed distances between the unknown shape and a model. A class division stops when that class contains maximum p^* models. N_C and N_C^* are computed with the following relations:

$$N_C = c_1 \cdot c_2 \cdot .... \cdot c_{L_{max}} \cdot p^*, \tag{3}$$

$$N_C^* = c_1 + c_2 + ... + c_{L_{max}} + p^*. \tag{4}$$

The same division factor $c_i = c$ is considered for all classification levels in Table 1.

Table 1

c	L_{max}	p^*	N_C^*	N_C
3	1	10	13	30
3	4	10	22	810
3	4	2	14	162
3	5	5	20	1215
5	3	5	20	625
5	3	2	17	162
10	3	2	32	2,000
10	4	2	42	20,000
10	4	10	50	100,000

The algorithm of Kim and Park is more efficient (faster) when the following values increase: L_{max}, p^* and/or c_i, $i = \overline{1, L_{max}}$.

3 PRINCIPLE OF THE SHAPE RECOGNITION METHOD

Modeling of the known shapes is presented in (Purcaru 2003). Let us consider d descriptors: s invariant and the others $(d - s)$ quasi-invariant to shape localization in the sensory space. Each known shape is v times explored, in the same conditions, and v vectors of the characteristic features result: $m_i^q = [m_{i,1}^q m_{i,2}^q ... m_{i,d}^q]^T$, $q = \overline{1, v}$, and $m_{i,j}^1 = m_{i,j}^2 = ... = m_{i,j}^v$ for $j = \overline{1, s}$.

One vector of the characteristic features, $m_i = [m_{i,1} m_{i,2} .... m_{i,d}]^T$, and the associated vector of the accepted tolerance, $\varepsilon_i = [\varepsilon_{i,1} \varepsilon_{i,2} ... \varepsilon_{i,d}]^T$, are defined in (Purcaru 2003) for each known shape (model):

$$m_{i,j} = \begin{cases} m_{i,j}^q, j = \overline{1, s}, 1 \leq q \leq v \\ \dfrac{1}{v} \sum_{t=1}^{v} m_{i,j}^q, j = \overline{s+1, d} \end{cases} \tag{5}$$

and

$$\varepsilon_{i,j} = \begin{cases} 0, j = \overline{1, s} \\ 3\sqrt{\dfrac{1}{v-1} \sum_{t=1}^{v} \left(m_{i,j}^q - m_{i,j}\right)^2}, j = \overline{s+1, d} \end{cases} \tag{6}$$

In conclusion, around a point $M_i(m_{i,1}, m_{i,2}, ... m_{i,d})$, corresponding to the model m_i, there is a region of the accepted tolerance $R_{at,i}$ in the d-dimensional descriptor space:

$$R_{at,i} = \left\{ x \middle\| x_j - m_{i,j} \middle| \leq \varepsilon_{i,j}, j = \overline{1, d} \right\}. \tag{7}$$

If an unknown shape x is inside $R_{at,i}$, that shape can be identified with the model m_i. The shape x, with

$$\left| x_j - m_{i,j} \right| \leq \varepsilon_{i,j}, 1 \leq j \leq d, \tag{8}$$

is considered similar with the model m_i, according to descriptor j. All models that satisfy this condition form the nearest neighbors of x according to descriptor j.

The shape recognition method, presented in this paper, supposes model classification and shape identification.

The model classification is realized only once, in the learning stage of the recognition process; this model sorting is realized at many levels, according to invariant descriptors.

The shape identification is realized in three stages:
- Identification of the most similar class;
- Identification of the nearest neighbors;
- Identification of the most similar model.

3.1 Model Classification

The invariant descriptors are used to classify the models, at h levels ($1 \le h \le s$) of classification.

♦ *Level 1*

The models are classified depending on the values $m_{i,1}$, $i = \overline{1, p}$. Each distinct value of the first descriptor generates a class C_i, $i = \overline{1, c_1}$ with $N(C_i)$ models.

The classification stops, for the models contained by C_i, if $N(C_i) \le p^*$, $1 \le i \le c_1$; otherwise the classification continues.

♦ *Level 2*

Each class C_i with $N(C_i) > p^*$ is divided in $c_{i,2}$ classes $C_{i,j}$, $j = \overline{1, c_{i,2}}$, depending on the values $m_{i,2}$ of the models contained by C_i; each distinct value of the second descriptor generates a class $C_{i,j}$, with $N(C_{i,j})$ models.

The classification stops, for the models contained by $C_{i,j}$, if $N(C_{i,j}) \le p^*$, $1 \le j \le c_{i,2}$; otherwise, the model classification continues.

♦ This model classification stops when
   a) each group of shapes, resulted at the last level of classification, contains maximum p^* models, or
   b) the procedure passes through s levels of classification.

♦ At the h level of classification, $1 \le h \le s$, each class is identified by the characteristic value $m_{p,h}$; all models contained by this class have the same value of the descriptor h.

♦ The models of the class $C_{i,i+1,...i+q}$ are similar in accordance to the first $(q+1)$ invariant descriptors.

3.2 Shape Identification

The class of the most similar with x models is firstly established. The nearest neighbors are then identified based on the quasi-invariant descriptors. The unknown shape can be identified with the most similar model that represents the nearest neighbor of x.

♦ *Identification of the most similar class*

The class C, that contains the most similar with x models, is identified after many stages.

The value x_1 establishes the class C_i (with the characteristic value $m_{i,1} = x_1$). The value x_2 establishes the class $C_{i,j}$ (with the characteristic value $m_{i,2} = x_2$) if $N(C_i) > p^*$. The class identification stops when all values $x_1, x_2, ... x_s$ are used or the last resulted class contains maximum p^* models.

If $N(C) > p^*$, the procedure continues with the identification of the nearest neighbors; otherwise, it continues with the identification of the most similar model.

♦ *Identification of the nearest neighbors*

Among the models of class C, the k-nearest neighbors are identified in many levels (stages), using the quasi-invariant descriptors. The values x_{s+q} and $m_{i,s+q}$ (for the models contained by the studied class) are represented on the axis of the descriptor $(s+q)$ for each level q of identification. The similarity (between x and each model) is then studied, according to this descriptor. The reference value

$$\varepsilon_{M,s+q} = \max_i \varepsilon_{i,s+q} \qquad (9)$$

is computed at each level, for the analyzed models. At the q level, the study starts with the nearest model and stops when
- there is not any model on the explored semi-axis or
- the condition
$$\left| x_{s+q} - m_{i,s+q} \right| > \varepsilon_{M,s+q} \qquad (10)$$
  is satisfied.

The k_q-nearest neighbors (established at the q level of identification) are the models similar with x, in accordance to descriptors $1, 2, ... (s+q)$. Such model m_i satisfies the condition

$$\left| x_j - m_{i,j} \right| \le \varepsilon_{i,j}, \ j = \overline{1, s+q} \qquad (11)$$

and form the class G_q, with $N(G_q) = k_q$. The neighbor identification stops if $k_q \le p^*$, and otherwise it continues at the level $(q+1)$.

The group, resulted at the last neighbor identification level, is denoted G and $N(G) = k$. The k-nearest neighbors are the models denoted $n_i = \left[n_{i,1} n_{i,2} ... n_{i,d} \right]^T$, $i = \overline{1, k}$.

♦ *Identification of the most similar model*

The unknown shape x can be identified with the model n_i if

$$\left| x_j - n_{i,j} \right| \le \varepsilon_{i,j}^{(n)}, \ 1 \le i \le k, \ j = \overline{1, d}, \qquad (12)$$

where $\varepsilon_i^{(n)}$ is the vector of the accepted tolerance associated with the model n_i. The condition (12) is

already verified for some descriptors, at the previously levels of identification.

4 EXPERIMENTAL RESULTS

Many shapes were recognized using the method presented in this paper. For example, the models are 60 square shapes ($p = 60$) with or without square holes; a model can contain maximum 6 holes. Let denote

- S_E and S_{Ht} the side of the exterior square and the side of the hole t respectively,
- S_E^* and S_{Ht}^* the relative (dimensionless) values of S_E and S_{Ht}.

The model distribution is presented in Tables 1, 2, 3, depending on N_H, S_E^* and S_{Ht}^*.

Table 1

N_H $\diagdown$ S_{Ht}^* $\diagdown$ S_E^*	0	1					
		1.5	3	4	5	7	8
1.5	*						
3	*						
4	*						
5	*	*					
7	*	*	*	*			
8	*	*	*	*	*		
9	*	*	*	*	*		
10	*	*	*	*	*	*	
11	*	*	*	*	*	*	*

Table 2

N_H $\diagdown$ S_{Ht}^* $\diagdown$ S_E^*	2				3		
	1.5 1.5	1.5 3	1.5 4	1.5 5	1.5 1.5 1.5	1.5 1.5 4	1.5 1.5 5
8	*				*		
9	*	*			*		
10	*	*	*		*	*	
11	*	*	*	*	*	*	*

Table 3

N_H $\diagdown$ S_{Ht}^* $\diagdown$ S_E^*	4			5			6	
	4× 1.5	4; 3× 1.5	5; 3× 1.5	5× 1.5	4; 4× 1.5	5; 4× 1.5	4; 5× 1.5	5; 5× 1.5
8	*							
9	*							
10	*							
11	*	*	*	*	*	*	*	*

The selected descriptors are the following:

- the number of the holes, $m_{i,1} = N_H$;
- the relative area of the shape limited by the exterior outline, $m_{i,2} = A_E^*$;

- the relative area of each hole contained by the explored shape, $m_{i,2+t} = A_{Ht}^*$, $t = \overline{1,6}$.

The first descriptor is invariant to shape localization in the sensory space, and $m_{i,3}, ..., m_{i,8}$ are quasi-invariant descriptors. So, $d = 8$ and $s = 1$. A model m_i, with N_H holes, has values zero for the last $(6 - N_H)$ descriptors.

The nominal values $m_{i,2}$ and the associated accepted tolerances $\varepsilon_{i,2}$ are specified in Table 4, for the models without holes.

Table 4

S_E^*	i	$m_{i,2}$	$\varepsilon_{i,2}$
1.5	1	2.2	1.265
3	2	8.9	2.625
4	3	16	1.999
5	4	24.7	2.469
7	5	48.7	5.109
8	6	63.6	4.05
9	7	81.2	4.17
10	8	101.5	4.527
11	9	121	3.162

The unknown shape $x = [1; 103; 53.5; 0; 0; 0; 0; 0]^T$ must be recognized.

The model classification is only depending on N_H; the resulted classes are C_i, $i = \overline{1,7}$. The number of models contained by each class is specified in Table 5.

Table 5

i	1	2	3	4	5	6	7
$N(C_i)$	9	23	10	7	6	3	2

Let consider $p^* = 3$. Because $x_1 = 1$, the most similar class with x is C_2, that contains 22 models.

The identification of the nearest neighbors starts with the study of similarity in accordance to the second descriptor, A_E^*. The models of the class C_2 have only 6 distinct values of A_E^* (24.7; 48.7; 63.6; 81.2; 101.5; 121) and the maximum value of their accepted tolerance is $\varepsilon_{M,2} = 5.109$. Only one value of A_E^* satisfies the condition (11) because $x_2 = 103$. So, all 5 models with $A_E^* = 101.5$, form the k_1-nearest neighbors of x.

The next level of neighbor identification studies the similarity in accordance to the third descriptor, A_{Ht}^*. The k_1-nearest neighbors have the following distinct values of A_{H1}^*: 2.2; 8.9; 16; 24.7; 48.7. Only the last value satisfies the condition (11) because $x_3 = 53.5$ and $\varepsilon_{M,3} = 5.109$. The shape x is identified with the model $m_{26} = [1; 101.5; 48.7; 0; 0; 0; 0; 0]^T$ because the condition (12) is satisfied.

5 CONCLUSION

This paper presents a method for the shape recognition, based on the identification of the nearest neighbors of the shape that must be recognized. Many invariant or quasi-invariant characteristic features assure the description of each analyzed shape. The recognition method supposes the classification of the models and the shape identification. This method has the following main advantages:

- assures a simple and fast recognition and good performance in practical applications;
- establishes the models the most similar with the unknown shape, at different levels of similarity;
- is adequate for the known shapes with spreads of their descriptor values;
- the unknown shape must be explored only once, regardless to its location in the sensory space.

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